Exam Answers, Labour Economics, June 2017

June 27, 2017

Question 1 - Unemployment search and activation ANSWERS

Q1: We differentiate equation $rV_u = b - s + \frac{\psi \frac{1}{\gamma} s^\gamma}{r} \int_x^\infty (1 - H(w))\, dw + \dot{V}_u$ with respect to s and obtain

$$
\begin{aligned}
1 &= \frac{\psi s^{\gamma - 1}}{r} \int_x^\infty (1 - H(w))\, dw \Leftrightarrow \\
s^{1-\gamma} &= \frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw \Leftrightarrow \\
s^* &= \left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw \right]^{\frac{1}{1-\gamma}}
\end{aligned}
$$

We need to check that s^*, in fact, maximizes the value of unemployment. To do this, we will examine the second-order derivative is negative

$$(\gamma - 1) \frac{\psi s^{\gamma - 2}}{r} \int_x^\infty (1 - H(w))\, dw$$

This is negative since $0 < \gamma < 1$.

Differentiating the optimal search effort s^* with respect to the reservation wage x yields

$$\frac{\partial s^*}{\partial x} = -\frac{1}{1-\gamma} \left(\frac{\psi}{r} \right)^{\frac{1}{1-\gamma}} \left[\int_x^\infty (1 - H(w))\, dw \right]^{\frac{\gamma}{1-\gamma}} (1 - H(x)) < 0$$

The intuition is that a higher reservation wage implies that the unemployed is willing to accept fewer jobs. Hence, the benefits of searching more is lower such that the optimal search effort is lower.

Differentiating the optimal search effort with respect to the search technology parameter ψ gives

$$
\begin{aligned}
\frac{\partial s^*}{\partial \psi} &= \frac{1}{1-\gamma} \left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw \right]^{\frac{\gamma}{1-\gamma}} \frac{1}{r} \int_x^\infty (1 - H(w))\, dw \\
&= \frac{1}{1-\gamma} \psi^{\frac{\gamma}{1-\gamma}} \left[\frac{1}{r} \int_x^\infty (1 - H(w))\, dw \right]^{\frac{1}{1-\gamma}} > 0
\end{aligned}
$$

Ceteris paribus, the better search technology (i.e. higher ψ), the higher search effort since the marginal benefits to search is increasing in ψ. Therefore, the worker would ceteris paribus search harder in the passive period, where she is not required to participate in active labor market policies. We stress that this result only holds for a given reservation wage and it could be the case that a higher search technology parameter, ψ, increases the reservation wage so much that the overall effect on the optimal search effort is negative.

Q2: We can write the reservation wage as

$$\begin{aligned}
x &= b - s + \frac{\frac{\psi}{\gamma} s^{\gamma}}{r} \int_x^\infty (1 - H(w))\, dw + \frac{1}{r}\dot{x} \\
&= b - \left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}} + \frac{\psi}{\gamma r}\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{\gamma}{1-\gamma}} \int_x^\infty (1 - H(w))\, dw + \frac{1}{r}\dot{x} \\
&= b - \left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}} + \frac{1}{\gamma}\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}} + \frac{1}{r}\dot{x} \\
&= b + \left(\frac{1}{\gamma} - 1\right)\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}} + \frac{1}{r}\dot{x} \\
&= b + \left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}} + \frac{1}{r}\dot{x}
\end{aligned}$$

If we focus on the stationary solution we set $\dot{x} = 0$ and obtain

$$x = b + \left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}}$$

Differentiating the stationery solution for the reservation wage with respect to ψ gives us

$$\begin{aligned}
\frac{\partial x}{\partial \psi} &= \frac{1}{1-\gamma}\left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{\gamma}{1-\gamma}} \frac{1}{r}\int_x^\infty (1 - H(w))\, dw \\
&\quad - \frac{1}{1-\gamma}\left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{\gamma}{1-\gamma}} \frac{\psi}{r}(1 - H(x))\frac{\partial x}{\partial \psi} \\
\frac{\partial x}{\partial \psi} &= \frac{\frac{1}{\gamma r}\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{\gamma}{1-\gamma}} \int_x^\infty (1 - H(w))\, dw}{1 + \frac{\psi}{\gamma r}(1 - H(x))\left[\frac{\psi}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{\gamma}{1-\gamma}}} > 0
\end{aligned}$$

Hence, a higher search technology parameter will increase the reservation wage. Therefore, we should expect a higher search effort in the passive period.

Q3: The differential equation is given by

$$\begin{aligned}
x(t) &= b + \left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\psi_p}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}} + \frac{1}{r}\dot{x}(t) \Leftrightarrow \\
\dot{x}(t) &= rx - rb - r\left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\psi_p}{r} \int_x^\infty (1 - H(w))\, dw\right]^{\frac{1}{1-\gamma}}
\end{aligned}$$

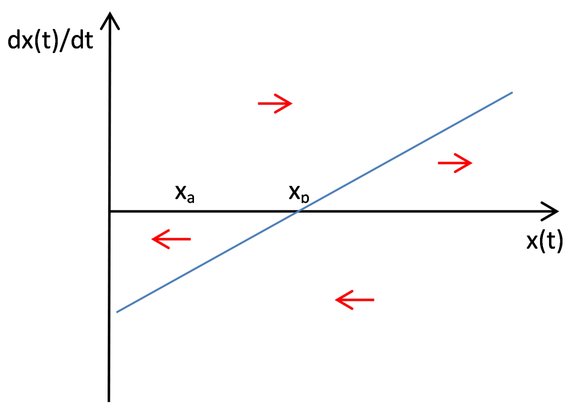
and the boundary condition for duration of P is given by

$$x(P) = b + \left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\psi_a}{r}\int_x^\infty (1 - H(w))\,dw\right]^{\frac{1}{1-\gamma}}$$

To determine whether the reservation wage increasing or decreasing until the end of the passive period, first, we need to differentiate $\dot{x}(t)$ with respect to $x(t)$. This gives us

$$\begin{aligned}\frac{\partial \dot{x}}{\partial x} &= r + \frac{r}{\gamma}\left(\frac{\psi}{r}\right)^{\frac{1}{1-\gamma}}\left[\int_x^\infty (1 - H(w))\,dw\right]^{\frac{\gamma}{1-\gamma}}(1 - H(x)) \\ &= r + \frac{\psi}{\gamma}\left[\frac{\psi}{r}\int_x^\infty (1 - H(w))\,dw\right]^{\frac{\gamma}{1-\gamma}}(1 - H(x)) > 0\end{aligned}$$

Consider the phase diagram below. We denote the stationary solutions for, respectively, the passive and active periods as x_p and x_a. The reservation wage for a newly unemployed $x(0)$ cannot be larger than x_p since then it will increase over time and we know that $x_a < x_p$ when the passive period ends. Furthermore, the reservation wage for a newly unemployed worker can neither be below x_a since then it would be decreasing even further. Hence, the reservation wage for a newly employed will be in between x_a and x_p and then decrease to x_a.



Q4: Since we argued in question 1, that $\frac{\partial s^*}{\partial x} < 0$, and since the reservation wage, x, is decreasing as we approach the end of the passive period, we must have that the search effort is increasing until $t = P$.

Q5: The hazard rate is $\phi(x(t), s^*(t)) = \psi\frac{1}{\gamma}s^*(t)^\gamma (1 - H(x(t)))$. Since the search effort is increasing as time goes by in the passive period the job arrival rate is increasing over time and since the reservation wage is declining over time, the share of jobs accepted is also increasing over time. Hence, the hazard rate will be increasing over time while in the passive period. In the empirical literature, this effect is often referred to as a motivation (or threat) effect of activation.

Q6: The search effort has risen during the passive period of unemployment, but we know that the static solution to the search effort is lower in the active period than in the passive period of unemployment. This implies that the search effort will drop discontinuously. This is in contrast to the reservation wage since $x = rV_u$ and the value of being unemployed cannot be jumping without stochastic shocks to the value of being unemployed.

Q7: From the questions above, we have established that $x_p(t) \geq x_a(t)$. As $x(t) = rV_u(t)$, it implies that the value of being unemployed in the passive period of unemployment is higher than the value of being unemployed in the active period of unemployment. Hence, from the perspective of an unemployed worker in the passive period, her value of unemployment must be increasing in the time until the active period begins. This is also clear from the fact that the reservation wage is decreasing as we approach the active period of unemployment. Hence, extending the duration of the passive period will increase the reservation wage in the passive period of unemployment, i.e. $\frac{\partial x(t)}{\partial P} > 0$ for $t \leq P$. We found in question 1 that a higher reservation wage implies a lower search effort, so we must have that $\frac{\partial s^*(t)}{\partial P} < 0$ for $t \leq P$. With the hazard rate defined as $\phi(x(t), s^*(t)) = \psi \frac{1}{\gamma} s^*(t)^\gamma (1 - H(x(t)))$ we have that a longer passive period implies a lower hazard rate, i.e. $\frac{\partial \phi(x(t), s^*(t))}{\partial P} < 0$.

Question 2 - Education subsidies ANSWERS

The phrasing of this question is supposed to be very open-ended. As a result, students do not need to follow exactly in the footsteps of the answers below to get full credit. Instead, credit should be awarded based on whether the students answers are correct and whether the approach to the question (including their modelling choices) demonstrate an understanding of the course material.

Q1: A simple model that can be used to analyze this question is the basic human capital model with general human capital. The economy consists of one (representative) worker and a number of firms. Time is continuous and infinite and agents discount the future at a rate r. The worker maximizes discounted lifetime income, while firms maximize profits. A worker with i units of human capital is assumed to produce $y(i)$ units of output at each point in time in which they work for a firm, where $y' > 0$ and $y'' < 0$. When not working, the worker earns z but we impose the assumption $y(0) > z$ to ensure that the worker will always work in equilibrium. Because human capital is general, the worker's output is the same at all firms, so all firms will be willing to offer a wage of up to $y(i)$ units of output. Accordingly, the worker's wage will be $y(i)$ at each point in time. Finally, we assume that worker must decide how much education to get, which in this model is simply the same as choosing how much general human capital to invest in, i. Following the description in the question, we abstract from the fact that education takes time but do include a cost of education, which we normalize to 1.

To solve for the level of education that the worker chooses, we first note that a worker who gets i units of education will earn a wage of $y(i)$ throughout his life. The worker's maximization problem is therefore:

$$\max_i \int_0^\infty y(i)e^{-rt}dt - i = \max_i \frac{y(i)}{r} - i$$

The first order condition for this problem is:

$$y'(i) = r$$

Since $y'' < 0$ this equation uniquely characterizes the level of education that workers will invest in, i.

Q2: We let s denote the education subsidy and let τ denote the level of the lumpsum tax paid at each point in time. Since workers take the lumpsum tax as given but now receive a subsidy for their education, the workers' problem is now:

$$\max_i \int_0^\infty (y(i) - \tau)e^{-rt}dt - (1-s)i = \max_i \frac{y(i) - \tau}{r} - (1-s)i$$

Wtiting up the first order conditions we get a new equation characterizing the level of education in equilibrium:

$$y'(i) = (1-s)r$$

Increasing the subsidy s lowers the right hand side of this equation and since $y'' < 0$ this implies that i must increase to make the left hand side smaller as well. So increases in the subsidy increase the level of education.

Q3: Since workers maximize discounted lifetime income, welfare maximization is simply the same as maximizing the present value of total future output net of education costs (how this output is divided is irrelevant). Since trivially it is optimal for the worker in the model to always be working, we can further focus on this case. The present value of total output net of education costs is then

$$Q = \int y(i)e^{-rt}dt - i = \frac{y(i)}{r} - i$$

We note that this is exactly what the worker maximizes in the absence of subsidies ($s = 0$). Accordingly, having no subsidies achieves the first best outcome and is economically efficient. The intuition behind this result is that in the current model, the returns to getting education and building human capital are captured entirely by the worker. Accordingly, when the worker also carries the cost of the education, he will balance the full costs and gains of additional education and choose the socially optimal level. Introducing education subsidies only distorts this choice by lowering the cost felt by the worker.

Q4: One simple way to generate differences in eduation levels is to introduce differences in inherent abilities as follows: We now assume that there are two different workers in the model, H and L who differ in their inherent ability level. We assume that these workers are the same in all ways except the amount of output they produce. When worker H has i_H units of human capital we assume that he produces $A_H y(i_H)$ units of output at each point in time. When worker L has i_L units of human capital we assume that he produces $A_L y(i_L)$ units of output. Here $A_H > A_L$ because H is high ability. Assuming firms are perfectly informed about these differences in output, the same reasoning as in Q1 implies that worker H will earn a wage of $A_H y(i_H)$, while worker L will earn $A_L y(i_L)$.

To solve the model we can look at the general optimization problem faced by some worker $j = H, L$ It is:

$$\max_{i_j} \int_0^\infty A_j y(i_j) e^{-rt} dt - i_j = \max_{i_j} \frac{A_j y(i_j)}{r} - i_j$$

Writing up the first order conditions as above, we arrive at the following two equations that characterize the optimal education choice of the two workers:

$$y'(i_H) = \frac{r}{A_H}$$

$$y'(i_L) = \frac{r}{A_L}$$

Because $A_H > A_L$, the right hand side of the equation for i_H is smaller than in the equation for i_L and because $y'' < 0$ this implies that $i_H > i_L$. So in the absence of education subsidies, the high ability worker H gets more education than the low ability worker L.

Q5: Introducing the education subsidy, the optimization problem faced by some worker $j = H, L$ becomes:

$$\max_{i_j} \int_0^\infty (A_j y(i_j) - \tau) e^{-rt} dt - (1-s) i_j = \max_{i_j} \frac{A_j y(i_j) - \tau}{r} - (1-s) i_j$$

The first order conditions then become:

$$y'(i_H) = \frac{(1-s)r}{A_H}$$

$$y'(i_L) = \frac{(1-s)r}{A_L}$$

Following the same arguments as in Q2 and Q4, we see that higher education subsidies lead both workers to get more education but that we will always have that the high ability worker gets more education than the low ability worker.

To examine economic efficiency we can again look at the discounted value of total output. Again relying on the fact that it is efficient for both workers to work at all times, the discounted value of total output net of education costs is now:

$$Q = \sum_{j=L,H} \left(\int_0^\infty A_j y(i_j) e^{-rt} dt - i_j \right) = \sum_{j=L,H} \left(\frac{A_j y(i_j)}{r} - i_j \right)$$

If we maximize this with respect to the amounts of education of the two workers, we get the following first order conditions:

$$y'(i_H) = \frac{r}{A_H}$$

$$y'(i_L) = \frac{r}{A_L}$$

These conditions coincide with the equations characterizing the workers' education choices without subsidies ($s = 0$). Thus, we again see that having no subsidies achieves the first best outcome and is economically efficient. The intuition is the same as in Q3. We have now introduced two workers with different productivities, however, they both still capture the full gains from additional education and human capital. Accordingly, when they are also faced with the full cost of the education they choose the socially optimal level.

Q6: Two realistic modifications are human capital externalities or capital constraints. Students obviously only need to discuss one (and should also get points if they point out other modifications as long as they are correct and reasonably realistic) but we cover both here for reference:

Human capital externalities can be incorporated by assuming that the output produced by one worker also depends on the human capital level of the other worker so that the output of worker H is $A_H(y(i_H) + f(y_L))$ when H has i_H units of human capital and L has i_L units. The output of worker L is $A_L y(i_L) + f(i_H))$. As before, we assume that $y' > 0$ and $y'' < 0$ so that a worker's output is increasing and concave in the workers own level of human capital but now we further have a function f where we assume $f' > 0$ and $f'' < 0$. This captures that one worker's output is also higher if the other worker has more human capital, for example, because the two workers need to work closely together in the economy.

Since wages reflect workers' own output, worker H's problem now becomes:

$$\max_{i_H} \int_0^\infty \left(A_H(y(i_H) + f(i_L)) - \tau\right) e^{-rt} dt - (1-s)i_H = \max_{i_H} \frac{A_H(y(i_H) + f(i_L)) - \tau}{r} - (1-s)i_H$$

The first order condition for worker H is unchanged however:

$$y'(i_H) = \frac{(1-s)r}{A_H}$$

Similar derivations for worker L also yields:

$$y'(i_L) = \frac{(1-s)r}{A_L}$$

As a result, the behavior of the workers is unchanged here and a higher subsidy s leads both workers to acquire more educaiton.

To examine efficiency we again focus on a planner's problem and look at the discounted value of total output:

$$
\begin{aligned}
Q &= \int_0^\infty A_H(y(i_H) + f(i_L))e^{-rt}dt + \int_0^\infty A_L(y(i_L) + f(i_L))e^{-rt}dt - i_H - i_L \\
&= \frac{A_H(y(i_H) + f(i_L))}{r} + \frac{A_L(y(i_L) + f(i_H))}{r} - i_H - i_L
\end{aligned}
$$

Next we look at the derivatives of this with respect to the education levels i_H and i_L:

$$
\begin{aligned}
\frac{\partial Q}{\partial i_H} &= \frac{A_H y'(i_H)}{r} + \frac{A_L f'(i_L)}{r} - 1 \\
\frac{\partial Q}{\partial i_L} &= \frac{A_L y'(i_L)}{r} + \frac{A_H f'(i_L)}{r} - 1
\end{aligned}
$$

We can evaluate these derivatives in the no subsidy outcome. From the workers' first order conditions we know that the outcome under no subsidies satisfies $y'(i_H) = \frac{r}{A_H}$ and $y'(i_L) = \frac{r}{A_L}$. Plugging this in we get:

$$
\begin{aligned}
\frac{\partial Q}{\partial i_H} &= 1 + \frac{A_H f'(i_L)}{r} - 1 = \frac{A_L f'(i_L)}{r} > 0 \\
\frac{\partial Q}{\partial i_L} &= 1 + \frac{A_L f'(i_L)}{r} - 1 = \frac{A_H f'(i_L)}{r} > 0
\end{aligned}
$$

We see that total discounted output (and therefore total welfare) is increasing in both education levels when evaluated at the outcome without education subsidies ($s = 0$). Since the education levels chosen by both workers are (always) increasing with the subsidy level s, this implies that the introduction of (at least) a small subsidy is welfare improving in this case. The intuition is that if one worker's human capital level affects the output of other workers, this is a positive externality that workers fail to take into account when choosing how much education to get so that they get too little education relative to the optimum. A positive subsidy can rectify this by increasing education levels.

Capital constraints can be incorporated by assuming that when making their human capital decisions, workers cannot freely borrow to pay the education cost but can at most spend (borrow) x units of output to cover education costs. With this change, workers still maximize the same objective function as in Q5, however, the maximization is now subject to a capital constraint that reflects that the worker has to be able to cover education costs, net of the subsidy. The maximization problem becomes:

$$\max_{i_j} \frac{A_j y(i_j)}{r} - (1-s)i_j - \tau$$
$$\text{s.t.} \quad (1-s)i_j \leqq x$$

The first order conditions are then:

$$y'(i_H) = (1+\lambda_H)\,\frac{(1-s)r}{A_H}$$

$$y'(i_L) = (1+\lambda_L)\,\frac{(1-s)r}{A_L}$$

Here λ_H and λ_L are Lagrange multipliers that are positive if the constraint is binding. When the constraints bind, the education levels are pinned by the constraints, $i_H = \frac{x}{1-s}$ and $i_L = \frac{x}{1-s}$. When the constraints do not bind, the education levels are pinned down by the first order conditions above. In both cases we see that education levels are increasing in the level of the subsidy, s. In the following, we will assume that x is such that the constraint binds for both workers when there is no subsidy $s = 0$.

Next we look at efficiency. Discounted value of total output is unchanged from Q5. The derivatives with respect to the education levels are:

$$\frac{\partial Q}{\partial i_H} = \frac{A_H y'(i_H)}{r} - 1$$
$$\frac{\partial Q}{\partial i_L} = \frac{A_L y'(i_L)}{r} - 1$$

We can evalute these derivatives in the no subsidy outcome. As we assumed above, the capital constraint binds in these cases so the first order conditions hold with λ_H and λ_L being positive so $y'(i_H) = (1+\lambda_H)\,\frac{(1-s)r}{A_H}$ and $y'(i_L) = (1+\lambda_L)\,\frac{(1-s)r}{A_L}$. If we plug this in, we get:

$$\frac{\partial Q}{\partial i_H} = \lambda_H > 0$$
$$\frac{\partial Q}{\partial i_L} = \lambda_L > 0$$

We see that total discounted output (and therefore total welfare) is increasing in both education levels when evaluated at the outcome without education subsidies ($s = 0$). Since the education levels chosen by both workers are (always) increasing with the subsidy level s, this implies that the introduction of (at least) a small subsidy is welfare improving in this case. The intuition is that when the capital constraint is binding, the productivity gains from education are high enough that it would be socially optimal for workers to get more education, only workers cannot do so because of their lack of capital. In this case an education subsidy is welfare improving because it alleviates the capital constraint.